

Starter r Q

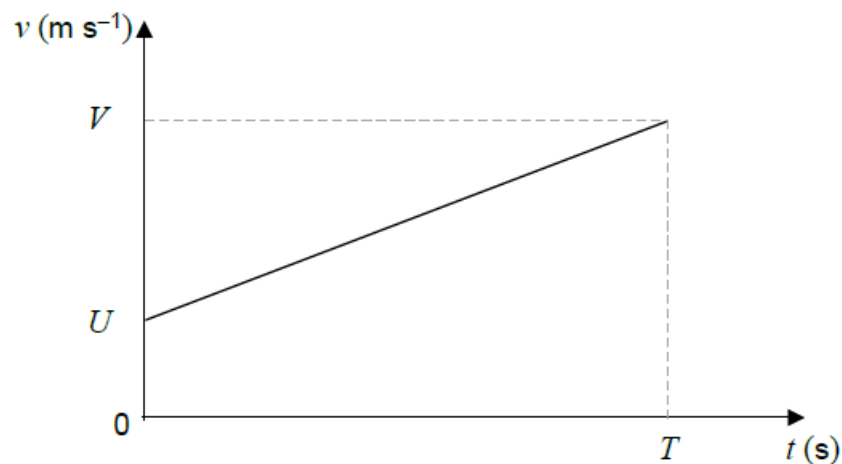
12

A particle moves on a straight line with a constant acceleration, $a \text{ m s}^{-2}$.

The initial velocity of the particle is $U \text{ m s}^{-1}$.

After T seconds the particle has velocity $V \text{ m s}^{-1}$.

This information is shown on the velocity-time graph.



The displacement, S metres, of the particle from its initial position at time T seconds is given by the formula

$$S = \frac{1}{2}(U + V)T$$

$$a = \frac{V - U}{T}$$

12 (a)

By considering the gradient of the graph, or otherwise, write down a formula for a in terms of U , V and T .

[1 mark]

Starter r Q

Need to eliminate T
from the equation
for S :

From (a):

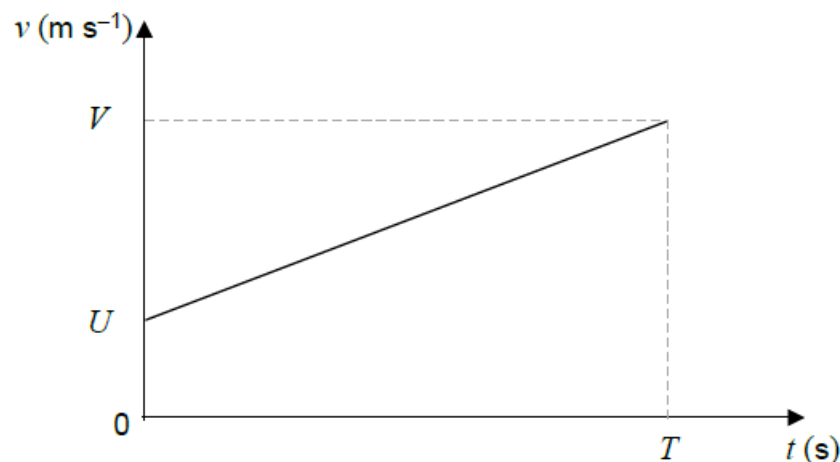
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The displacement, S metres, of the particle from its initial position at time T seconds is given by the formula

$$S = \frac{1}{2}(U + V)T$$

12 (b) Hence show that $V^2 = U^2 + 2aS$

[3 marks]

Starter Question MS

12(a)	States correct expression for a	AO1.1b	B1	$a = \frac{V-U}{T}$
(b)	Rearranges to make T the subject of the formula	AO2.1	R1	$T = \frac{V-U}{a}$
	Uses given expression for S and attempts to eliminate T	AO2.1	R1	$S = \frac{1}{2}(U+V) \times \frac{V-U}{a}$ $2as = (U+V)(V-U)$
	<p>Completes argument to reach required result AG</p> <p>Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips</p>	AO2.1	R1	$V^2 = U^2 + 2aS$ <p>(AG)</p>
	Total		4	

Q

Kinematics

Q3

Understand, use and derive the formulae for constant acceleration for motion in a straight line; extend to 2 dimensions using vectors.

Assessed at AS and A-level

Teaching guidance

Students should be able to:

- recall and use the following formulae:

$$v = u + at$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Note: the less fashionable $s = vt - \frac{1}{2}at^2$ is not essential.

7.3 Equations of motion for constant acceleration

Example 7 from last lesson

Car P is accelerating at 2ms^{-2} . When its velocity is 10ms^{-1} , it is overtaken by car Q, which is travelling at 16ms^{-1} and accelerating at 1ms^{-2} .

How long will P take to catch up with Q?
When P meets Q, their displacements must be the same at the same time (otherwise they don't meet)...

Car P:

$$u_p = 10\text{ms}^{-1}$$

$$16\text{ms}^{-1}$$

$$a_p = 2\text{ms}^{-2}$$

2

Car Q:

$$u_q =$$

$$a_q = 1\text{ms}^{-2}$$

Using

Car P:

7.3 Equations of motion for constant acceleration

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Using

Car Q:

7.3 Equations of motion for constant acceleration

Example 7 from last lesson

When P meets Q, $s_p = s_Q$:

Hence, when $t = 0\text{s}$, this is at the start when Q passes P, so P catches Q when $t = 12\text{s}$.

P1

Understand and use fundamental quantities and units in the SI system: length, time, mass.

Understand and use derived quantities and units: velocity, acceleration, force, weight, moment.

- understand that weight is a force, $W = mg$ (N)
- know that g is acceleration due to gravity (m s^{-2}) and in questions where a value of g is given, the final answer should be given to the same degree of accuracy as the value of g

R3

Understand and use weight and motion in a straight line under gravity; gravitational acceleration, g , and its value in SI units to varying degrees of accuracy.

(The inverse square law for gravitation is not required and g may be assumed to be constant, but students should be aware that g is not a universal constant but depends on location).

Assessed at AS and A-level

Teaching guidance

Students should be able to:

- understand the distinction between mass and weight
- state necessary modelling assumptions, and relate these to the model used.

R3

Understand and use weight and motion in a straight line under gravity; gravitational acceleration, g , and its value in SI units to varying degrees of accuracy.

(The inverse square law for gravitation is not required and g may be assumed to be constant, but students should be aware that g is not a universal constant but depends on location).

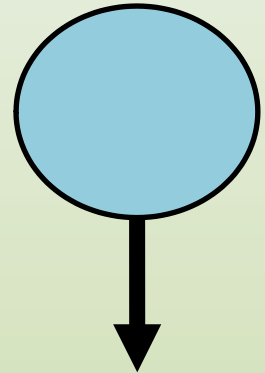
Notes

- In questions where a numerical value for g is needed, students will be clearly told which approximation to use and their answers must then be given to the same accuracy: 10 requires 1 significant figure, 9.8 two significant figures and 9.81 three significant figures.
- In questions involving objects in motion under gravity it will be assumed that:
 - g remains constant
 - objects can be treated as particles
 - resistance forces are negligible.

8.3 Motion under gravity

If an object is dropped, *ignoring the effects of air resistance*, the object will accelerate downwards due to its weight

acceleration due to gravity



Weight

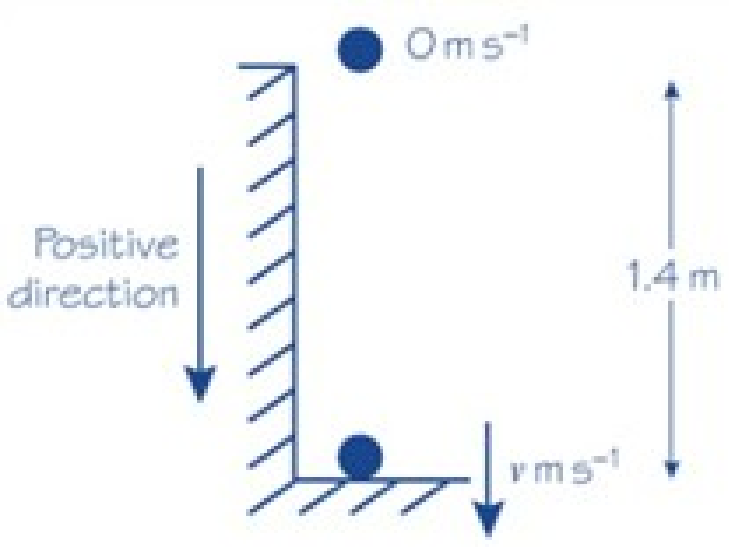
As gravity is constant, we can use the constant acceleration equations.

Use the same number of significant figures in your answer as the value of that you are given.

8.3 Motion under gravity

Example 1a

A book falls off the top shelf of a bookcase. The shelf is 1.4 m above a wooden floor. Find:
a the time the book takes to reach the floor, **b** the speed with which the book strikes the floor.



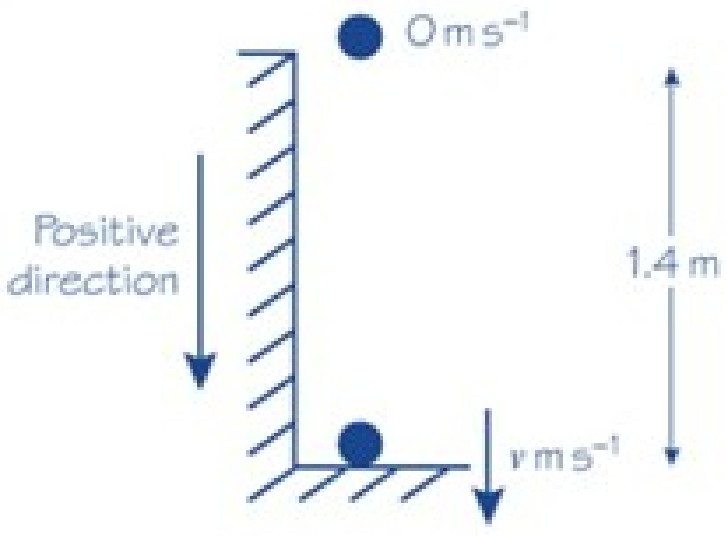
$$\therefore t = 0.5345 \dots$$

The time taken is 0.53 seconds to 2sf.

8.3 Motion under gravity

Example 1b

A book falls off the top shelf of a bookcase. The shelf is 1.4 m above a wooden floor. Find:
a the time the book takes to reach the floor, **b** the speed with which the book strikes the floor.

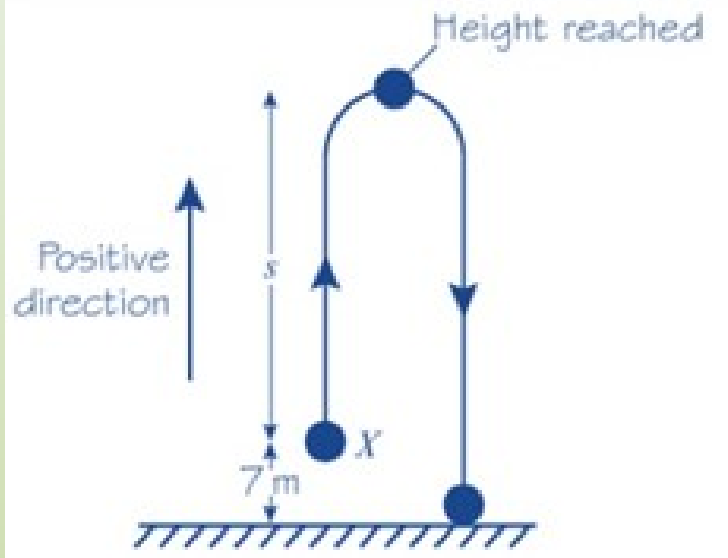


The speed with which the book strikes the floor is 5.2 m/s to 2sf.

8.3 Motion under gravity

Example 2a

A ball is projected vertically upwards, from a point X which is 7 m above the ground, with speed 21 m s^{-1} . Find: **a** the greatest height above the ground reached by the ball, **b** the time of flight of the ball.

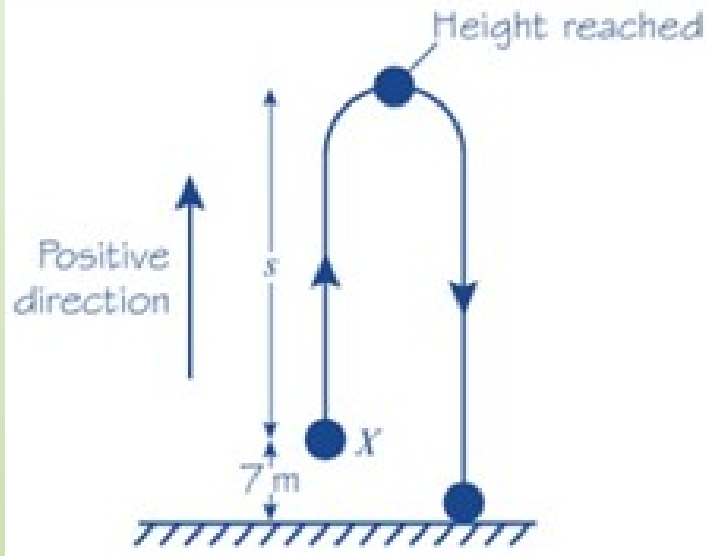


The greatest height is $22.5 + 7 = 29.5\text{m}$ 30m to 2sf.

8.3 Motion under gravity

Example 2b

A ball is projected vertically upwards, from a point X which is 7 m above the ground, with speed 21 m s^{-1} . Find: **a** the greatest height above the ground reached by the ball, **b** the time of flight of the ball.



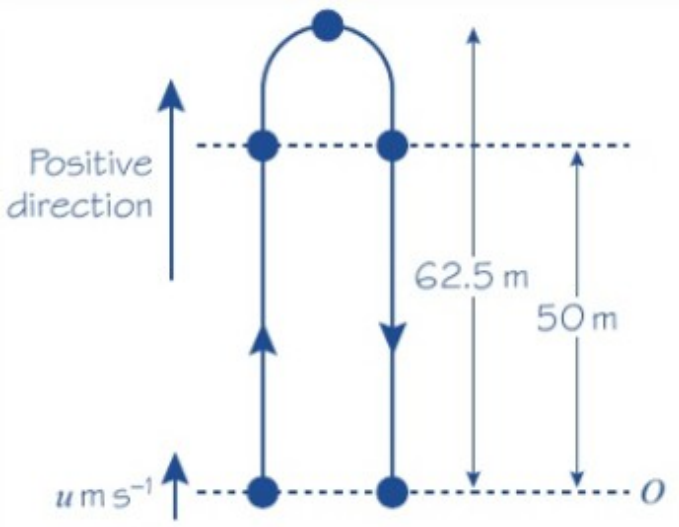
The time of flight is the total time that the ball is in motion from the time that it is projected to the time that it stops moving. Here the ball will stop when it hits the ground. The point where the ball hits the ground is 7 m below the point from which it was projected so $s = -7$.

The time of flight is 4.6 seconds to 2sf.

8.3 Motion under gravity

Example 3a

A particle is projected vertically upwards from a point O with speed $u \text{ m s}^{-1}$. The greatest height reached by the particle is 62.5 m above O . Find: **a** the value of u , **b** the total time for which the particle is 50 m or more above O .

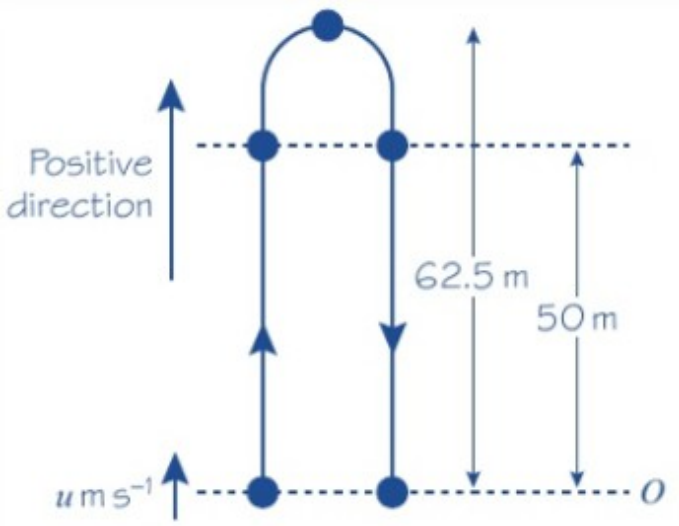


The initial velocity is 35m/s.

8.3 Motion under gravity

Example 3b

A particle is projected vertically upwards from a point O with speed $u \text{ m s}^{-1}$. The greatest height reached by the particle is 62.5 m above O . Find: **a** the value of u , **b** the total time for which the particle is 50 m or more above O .

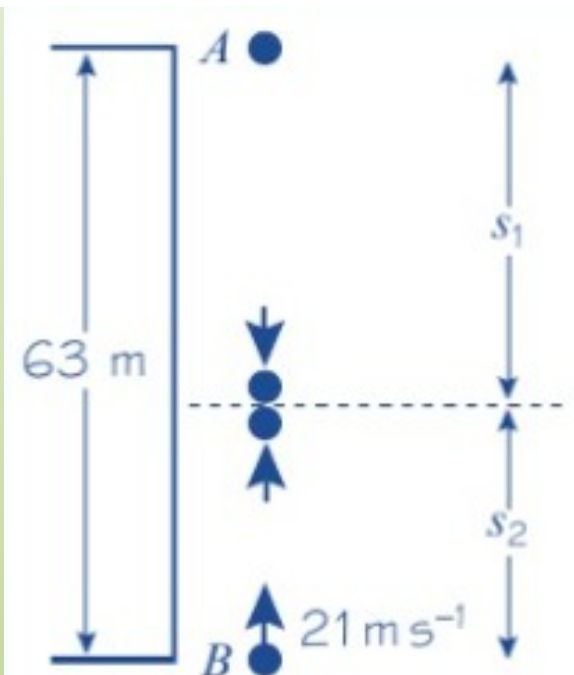


The particle is 50m or more above O for 2 seconds to 2sf.

8.3 Motion under gravity

Example 4 Take the positive direction to be downwards...

A ball A falls vertically from rest from the top of a tower 63 m high. At the same time as A begins to fall, another ball B is projected vertically upwards from the bottom of the tower with speed 21 m s^{-1} . The balls collide. Find the distance of the point where the balls collide from the bottom of the tower.



For A:

For B:

Acceleration is negative for B as it is travelling upwards and the positive direction is downwards.

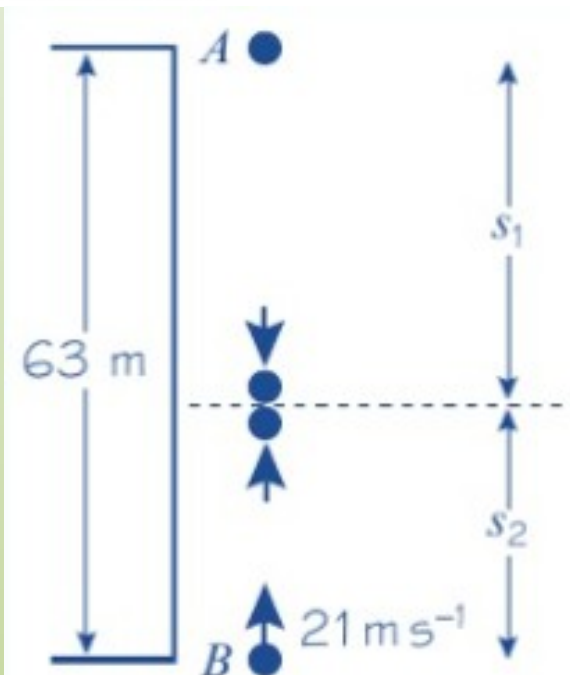
$$s_B = 21t - 4.9t^2$$

$$s_A + s_B = 63$$

8.3 Motion under gravity

Example 4 *Take the positive direction to be downwards...*

A ball A falls vertically from rest from the top of a tower 63 m high. At the same time as A begins to fall, another ball B is projected vertically upwards from the bottom of the tower with speed 21 m s^{-1} . The balls collide. Find the distance of the point where the balls collide from the bottom of the tower.



$$4.9 t^2 + 21 t - 4.9 t^2 = 63$$

They collide after 3 seconds but we need to know the distance from the bottom... i.e. what is when ?

$$s_B = 21 (3) - 4.9 (3)^2 = 18.9 \text{ m} \quad \text{to 2sf}$$